Optimum Thickness of Pressure-Sensitive Paint for Unsteady Measurements

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When pressure-sensitive paint (PSP) is used to measure unsteady pressures, the thickness of the paint layer critically affects both the PSP signal-to-noise ratio (SNR) and frequency response. As the thickness of a paint layer increases, the brightness of the signal, and thus the SNR, increases, whereas the frequency response decreases, resulting in attenuation of the unsteady component of the signal. In addition, if pressure fluctuations and frequencies are large and the paint layer is thick, the unsteady PSP signal may be distorted and the steady component may be offset from its true mean value. These observations suggest that, for each application, there is an optimum paint thickness where the "unsteady SNR" (the ratio of unsteady signal amplitude to steady noise) is a maximum and where the offset and distortion are small. This hypothesis is explored by numerically evaluating published analytic solutions to a simplified model of the unsteady PSP problem. It was found that over a wide range of parameters the unsteady SNR is a maximum when the paint thickness corresponds to a 1.25 dB attenuation of the unsteady signal. At this optimum, the nondimensional time constant ($h^2\omega/D$) is approximately 1.5.

Nomenclature

A = absorption per unit depth of incident photons by luminophore
a = coefficient in Stern–Volmer equation
b = coefficient in Stern–Volmer equation, $kS\chi$
c = concentration of luminophore
D = diffusivity of oxygen in paint binder, $\mu\text{m}^2/\text{s}$
E = incident light intensity
e = luminescence emission per unit depth
f = frequency, Hz
h = thickness of paint layer, μm
I = luminescence light intensity
k = Stern–Volmer coefficient
N = noise
n = oxygen concentration
p = pressure
r = amplitude ratio, p'_{psp}/p'
S = solubility of oxygen in binder
T = temperature
t = time
z = distance from paint–air interface, positive toward model
γ = $h\sqrt{(\omega/D)}$
δ = optical density, $c\sigma h/\ln(10)$
η = z/h
σ = effective absorption cross section of luminophore
τ = time constant, h^2/D, s
Φ = quantum yield
ϕ = phase shift, deg
χ = mole fraction of oxygen, 0.21 in air
ω = circular frequency, rad/s

Subscripts

psp = indicated by pressure-sensitive paint
ref = reference conditions
0 = value at paint–air interface, where $z = 0$

Superscripts

′ = unsteady component
‾ = steady component

Introduction

PRESSURE-SENSITIVE paint (PSP) has been widely used to measure steady pressure distributions on wind-tunnel models.[1−3] This technique requires coating the model with paint consisting of luminescent molecules (luminophore) suspended in an oxygen-permeable matrix or binder. The paint is illuminated with light of a frequency that excites the luminophores, and the brightness of the resulting luminescence is recorded with a camera. The luminescence intensity is inversely proportional to the local pressure at the surface of the paint because oxygen in the air diffuses into the paint layer and, by absorbing excess energy from the excited luminophores, inhibits or "quenches" luminescence. The brightness of the luminescence also depends on the number of luminescent molecules in the paint, which is directly proportional to the thickness of the paint layer, and on the intensity of the incident illumination. Thus, brighter signals and higher signal-to-noise ratios (SNRs) can be achieved from thicker paint layers. In practice, the effects of point-to-point variations in paint thickness and incident light intensity are equalized by dividing each wind-on image by a reference image acquired at a constant, wind-off pressure.

The dynamic response of pressure paint is dominated by the diffusion of oxygen through the binder. A detailed description of oxygen diffusion in polymer binders and luminescence quenching is provided by Lu and Winnik.[4] Using a simplified physical model, researchers in Russia[5,6] and at the University of Florida[7,8] independently derived analytic solutions for unsteady PSP responses to step and sinusoidal pressure changes. They showed that the time required for PSP to respond to a step change in pressure is directly proportional to the square of the thickness of the paint layer and inversely proportional to the diffusivity of oxygen in the binder. For a sinusoidally varying pressure, they showed that, as frequency or thickness increases, the unsteady component of a PSP signal is increasingly attenuated and phase shifted. Winslow et al.[9] showed that, as the input frequency increases beyond the "break" frequency (where the attenuation is 3 dB), PSP behaves as a one-half-order dynamic system with a rolloff of −10 dB/decade and asymptotic phase shift of −45 deg. Winslow et al.[10] compared solutions for linear and nonlinear paint calibrations and showed that the more realistic nonlinear calibration does a much better job of approximating the response of PSP to a positive step in pressure.

The tradeoff with thickness between signal strength and frequency response, that is, weak signals but fast response for thin
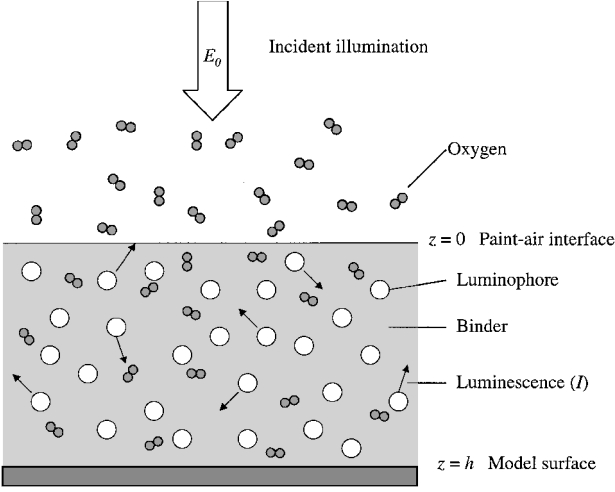
Fig. 1   Physical model of PSP.

layers vs strong signals but slow response for thick layers, invites the question: How thick should a paint layer be for a particular application? In this paper, it is proposed that an optimum thickness occurs where the ratio of unsteady signal amplitude to steady noise is a maximum. This optimum has been determined by numerically evaluating the analytic solutions of Mosharov et al.[5] for the PSP response to a sinusoidal applied pressure. These solutions have also been used to explore nonlinear effects and how they can distort the PSP signal.

Physical Model and Analytic Solutions

The results presented here were computed from analytic solutions for PSP dynamic response published by Mosharov et al.,[5] which, for completeness, will be recapitulated. The physical model (Fig. 1) consists of a paint layer of thickness h in which a luminophore is uniformly distributed with concentration c in a transparent, oxygen-permeable binder. It is assumed that the paint layer is applied directly to the surface of the test article, which does not reflect light and is impermeable to oxygen. This is a simplistic model because, in practice, the paint layer is usually applied on top of a white base coat that is diffusely reflective and may be permeable to oxygen. Nonetheless, this paper will first show results for the simplified model and then discuss how the results might be altered by the base coat.

A unit area of the paint layer is illuminated at normal incidence by light of constant intensity E_0, and the air pressure above the layer, $p(t)$, varies as a function of time. As incident photons pass through the layer, they are absorbed by the luminophore in direct proportion to the intensity $E(z)$, the concentration of luminophore, c, and the absorption cross section of the luminophore, σ. As a result of this absorption, the incident light is attenuated exponentially with depth in accordance with the Beer–Lambert law, $E(z) = E_0 e^{-\sigma c z}$, so that the absorption per unit depth at depth z is given by $A(z) = \sigma c E(z) = \sigma c E_0 e^{-\sigma c z}$.

The instantaneous luminescence observed from outside the layer, $I(t)$, is the sum of luminescence originating from all levels within the layer. The emission per unit depth at depth z and time t is given by the product of the absorption and the fraction of excited luminophores that luminesce [the local quantum yield, $\Phi(z, t)$] $e(z, t) = A(z)\Phi(z, t)$. When constant temperature and a linear model for the sorption of oxygen within the binder (Henry's law) are assumed, the quantum yield is inversely proportional to the local instantaneous concentration of oxygen according to the Stern–Volmer equation: $\Phi_{\text{ref}}/\Phi(z, t) = a + k \cdot n(z, t)$. (Note that k is a coefficient that depends on both the pressure and temperature reference conditions. In the literature, this coefficient is called the Stern–Volmer constant when the reference condition is vacuum, that is, $k = k_{\text{vac}}$.) When these expressions are combined, the instantaneous emission per unit depth from luminophores at depth z is given by

$$e(z, t) = A(z)\Phi(z, t) = \sigma c E_0 e^{-\sigma c z}\{\Phi_{\text{ref}}/[a + kn(z, t)]\}$$

When no absorption of luminescence is assumed, luminescence intensity at the surface is obtained by integrating across the depth of the paint layer:

$$I(t) = \frac{1}{2}\int_0^h e(z, t)\,\mathrm{d}z = \frac{1}{2}\sigma \cdot c \cdot E_0 \Phi_{\text{ref}} \int_0^h \frac{e^{-\sigma c z}}{a + kn(z, t)}\,\mathrm{d}z \tag{1}$$

The factor $\frac{1}{2}$ appears because half of the luminescence is directed toward the substrate, where it is absorbed.

When the temperature is assumed to be uniform over the thickness of the paint layer ($\partial T/\partial z = 0$), the variation of oxygen concentration within the layer is described by the one-dimensional diffusion equation

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2}$$

At the paint–air interface ($z = 0$), the boundary condition is $n(0, t) = S\chi p(t)$ in accordance with Henry's law. At the bottom of the layer ($z = h$), where the substrate is impermeable, the gradient of the oxygen concentration normal to the boundary must vanish:

$$\left.\frac{\partial n}{\partial z}\right|_{z=h} = 0$$

If the applied pressure is quasi steady, the oxygen concentration and quantum yield are uniform through the layer, and the signal intensity at the surface is

$$I(t) = \frac{1}{2}\frac{\Phi_{\text{ref}}}{a + kn(t)}E_0(1 - e^{-\sigma c h}) = \frac{1}{2}\frac{\Phi_{\text{ref}}}{a + bp(t)}E_0(1 - 10^{-\delta}) \tag{2}$$

where $\delta = \sigma c h/\ell n\,10$, $b \equiv kS\chi$, and Henry's law has been applied to convert $n(t)$ to $p(t)$. The indicated pressure is then equal to the true pressure:

$$p_{\text{psp}}(t) \equiv \{[I_{\text{ref}}/I(t)] - a\}/b = p(t) \tag{3}$$

If the pressure changes rapidly enough, however, the spatial variation of the oxygen concentration through the layer will be significant, and Eq. (3) will yield an indicated pressure that differs from the true pressure. If $p(t)$ varies sinusoidally with time, $p(t) = \bar{p} + p' \cdot \sin \omega t$, the indicated PSP pressure is given by[5]

$$p_{\text{psp}}(t) = (1 - 10^{-\delta})\bigg/ \left\{b\int_0^1 \frac{\delta \cdot 10^{-\delta \cdot \eta} \cdot \ell n(10)}{a + b\bar{p} + bp'[X(\gamma, \eta) \cdot \sin(\omega t) + Y(\gamma, \eta) \cdot \cos(\omega t)]}\,\mathrm{d}\eta\right\} - \frac{a}{b} \tag{4}$$

where

$$X(\gamma, \eta) = \frac{\cosh[\sqrt{2}\gamma(1 - \eta/2)] \cdot \cos[\gamma\eta/\sqrt{2}] + \cos[\sqrt{2}\gamma(1 - \eta/2)] \cdot \cosh[\gamma\eta/\sqrt{2}]}{\cosh[\sqrt{2}\gamma] + \cos[\sqrt{2}\gamma]}$$

$$Y(\gamma, \eta) = \frac{\sinh[\sqrt{2}\gamma(1 - \eta/2)] \cdot \sin[\gamma\eta/\sqrt{2}] + \sin[\sqrt{2}\gamma(1 - \eta/2)] \cdot \sinh[\gamma\eta/\sqrt{2}]}{\cosh[\sqrt{2}\gamma] + \cos[\sqrt{2}\gamma]}$$

Although the diffusion equation is linear, the PSP response is nonlinear because of the inverse relationship between intensity and oxygen concentration in the Stern–Volmer equation. However, if the amplitude of the unsteady pressure is small compared to the steady pressure ($p' \ll \bar{p}$), the solution can be linearized, and the indicated pressure becomes[5]

$$p_{psp}(t) = \bar{p}_{psp} + p'_{psp} \cdot \sin(\omega t + \phi)$$

$$\cong \bar{p} + p' \cdot \frac{\delta \cdot \ell n(10)}{1 - 10^{-\delta}} \int_0^1 10^{-\delta \cdot \eta}[X(\gamma, \eta) \cdot \sin(\omega t)$$

$$+ Y(\gamma, \eta) \cdot \cos(\omega t)]\, d\eta \qquad (5)$$

In this case, the indicated pressure is sinusoidal and has the same frequency as $p(t)$; however, its amplitude, p'_{psp}, is attenuated and its phase ϕ is shifted relative to the applied pressure. The unsteady amplitude ratio ($r = p'_{psp}/p'$) and phase shift (ϕ equals measured minus true) are given by

$$r = \sqrt{\alpha^2 + \beta^2} \qquad (6)$$

$$\phi = \tan^{-1}(\beta/\alpha) \qquad (7)$$

where

$$\alpha(\gamma) = \frac{\delta \cdot \ell n(10)}{(1 - 10^{-\delta})} \int_0^1 10^{-\delta \cdot \eta} X(\gamma, \eta)\, d\eta \qquad (8)$$

$$\beta(\gamma) = \frac{\delta \cdot \ell n(10)}{(1 - 10^{-\delta})} \int_0^1 10^{-\delta \cdot \eta} Y(\gamma, \eta)\, d\eta \qquad (9)$$

The amplitude ratio and phase shift completely describe the PSP response in the linearized case.

Results

Steady Response

Figure 2 presents the paint brightness ratio $\bar{I} = I/E_0\Phi = (1 - 10^{-\sigma c h/\ell n 10})$ as a function of thickness for a range of unit optical densities [$= \delta/h = \sigma c/\ell n(10)$] typical of pressure paints. When the optical density is large, most of the incident photons are absorbed, and the brightness ratio approaches 1.0. In this case, increasing the thickness of the layer has very little effect on the brightness of luminescence, and the layer is optically thick. Therefore, $I/E_0\Phi$ expresses the brightness of a layer of thickness h as a fraction of the brightness of an optically thick layer under the same conditions. For thin layers, the brightness ratio increases in nearly direct proportion to thickness. The rate of increase in brightness, however, decreases as the layer becomes thicker until, for optically thick layers in which nearly all of the incident light has been absorbed, it approaches zero. For paint of a fixed thickness, the brightness increases as the unit optical density increases, as would occur, for example, if the concentration of luminophore, c, were increased. In practice, adverse interactions among luminophores at high concentrations[11] cause deviations from the Bee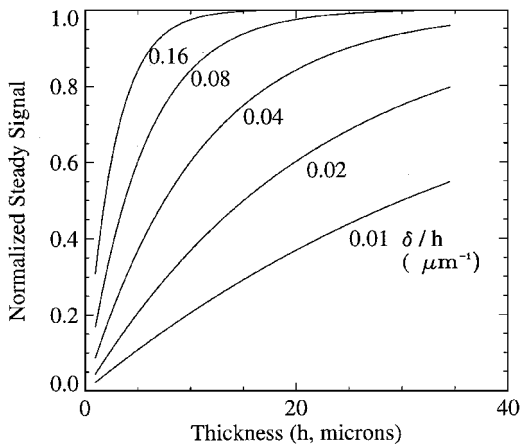r–Lambert law that limit the gains in brightness that can be achieved by increasing c. Eventually, these adverse effects become so large that brightness actually decreases as c increases.[12] For a widely used paint, PtTFPP in FIB (Ref. 13), peak brightness occurs at roughly $c = 10^{-2}$ mol/l. This corresponds to a unit optical density of $\delta/h = c\sigma/\ell n 10 \approx 0.13\,\mu m^{-1}$ when $\sigma/\ell n 10 \approx 1.3 \times 10^5$ mol^{-1}l·cm^{-1} (Ref. 14).

For maximum signal strength, the paint layer should be optically thick. This may not be permissible in practice, however, if the optically thick layer is so thick that it alters the aerodynamic shape of the model. Allowable paint thckness depends on the scale of the model and the test conditions.[15] For tests in the NASA Ames 12-Ft Pressure Wind Tunnel, paint layers have typically been 25–50 μm thick, far less than optically thick.

Unsteady Response: Small Perturbation Approximation

To explore the dynamic response of pressure paint, a computer program was written to integrate numerically Eqs. (8) and (9), and the solutions were used to compute r and ϕ from Eqs. (6) and (7). Figure 3 shows results for the amplitude ratio as a function of thickness over a range of frequencies. Values of unit optical density and diffusivity typical of current PSPs were assumed ($\delta/h = 0.01\,\mu m^{-1}$ and $D = 10^3\,\mu m^2$/s). The amplitude ratio decreases from 1.0 (the attenuation of the unsteady signal increases) as both paint thickness and input frequency increase. For very low frequencies, r approaches 1.0 even for thick layers, that is, the response becomes quasi steady. These results are essentially the same as those presented by Mosharov et al.[5] (their Fig. 2.3) except that they plot r vs a nondimensional frequency with optical density as a parameter. The amplitude ratio corresponding to 1.25 dB ($r = 0.866$) attenuation is indicated in Fig. 3 for future reference.

In the small perturbation approximation, the amplitude of the unsteady signal is proportional to the product of the amplitude ratio r (Fig. 3) and the steady signal amplitude $\bar{I}$ (Fig. 2). Figure 4
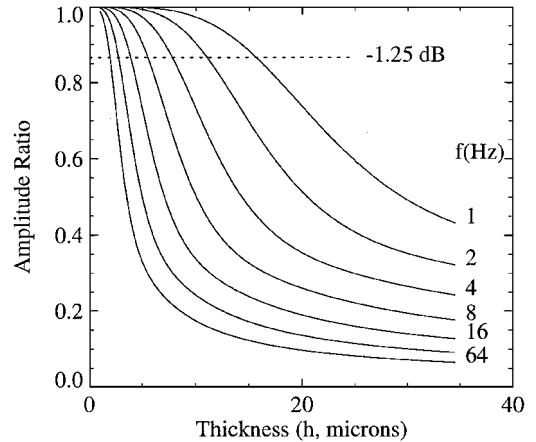
Fig. 3 Amplitude ratio vs thickness: $\delta/h = 0.01\ \mu m^{-1}$ and $D = 10^3\ \mu m^2$/s.

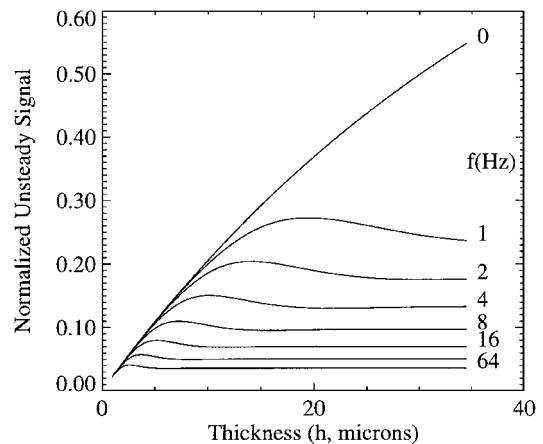Fig. 4 Normalized unsteady signal vs thickness: $\delta/h = 0.01\ \mu m^{-1}$ and $D = 10^3\ \mu m^2$/s.

Fig. 2 Normalized steady signal intensity vs thickness (Beer–Lambert law).

shows this product, $I' = \bar{I} \cdot r$, as a function of paint thickness with frequency of the applied pressure as a parameter (unit optical density and diffusivity are the same as in Fig. 3 and will be carried forward subsequently). I' is the actual unsteady signal amplitude as a fraction of the maximum possible unsteady amplitude, which occurs for an optically thick layer under quasi-steady conditions ($r = 1.0$ and $\bar{I} = 1.0$). I' increases approximately linearly with paint thickness for very thin paint layers. It then passes through a flat maximum and quickly approaches an asymptote that is not much less than the maximum. This occurs because of the offsetting effects of the increasing signal intensity and the decreasing amplitude ratio. The unsteady signal approaches its asymptote long before the paint has become optically thick. In a sense, the paint becomes pneumatically thick: the unsteadiness of the oxygen concentration does not have time to penetrate farther than a certain depth within the paint layer, and so the contribution to the unsteady signal of luminophores deeper than this depth is very small. At very low frequencies, I' approaches the quasi-steady limit. As frequency increases, the maximum in I' decreases and occurs at lower and lower thicknesses. Even at the lowest nonzero frequency shown (1 Hz), the maximum in I' is less than 30% of the value for an optically thick layer under quasi-steady conditions.

The unsteady signal amplitude (Fig. 4) is the numerator of the "unsteady SNR." The denominator is the steady signal noise, which, for this analysis, is assumed to be dominated by photon shot noise and to increase in proportion to the square root of the steady signal intensity ($N \propto \sqrt{\bar{I}}$). The normalized unsteady signal amplitude I', the steady signal noise N, and their quotient ($\mathrm{SNR}' = I'/N = r \cdot \bar{I}/\sqrt{\bar{I}} = r \cdot \sqrt{\bar{I}}$) are shown as functions of thickness in Fig. 5 for the case of a 4-Hz applied pressure. SNR' is the actual unsteady SNR expressed as a fraction of the maximum possible unsteady SNR, which corresponds to an optically thick layer under quasi-steady conditions. Note that SNR' first increases with paint thickness, then passes through a well-defined maximum, and finally decreases and continues to decrease with increasing paint thickness. The decrease in SNR' beyond the maximum occurs because noise continues to increase with increasing thickness, whereas I' is approximately constant. In Fig. 6, SNR' is shown as a function of paint thickness for a range of frequencies between 0 and 256 Hz. As frequency increases, the maximum in SNR' decreases, and the corresponding optimum thickness becomes smaller and smaller. For an applied frequency of 256 Hz, the optimum is only about 2 μm, a very thin paint layer. In addition, the peak becomes narrower with increasing frequency, and so the absolute margin for error in applying the optimum thickness of paint becomes smaller with increasing frequency.

Empirical observation revealed that, when the thicknesses shown in Fig. 6 are normalized by the thickness that corresponds to 1.25-dB attenuation (Fig. 3), the peaks in the SNR's all occur at a thickness ratio of about 1.0 (Fig. 7). At this optimum, the normalized time constant, $\tau\omega = (h^2/D)\omega = \gamma^2$, is approximately 1.48. That $\tau\omega$ is
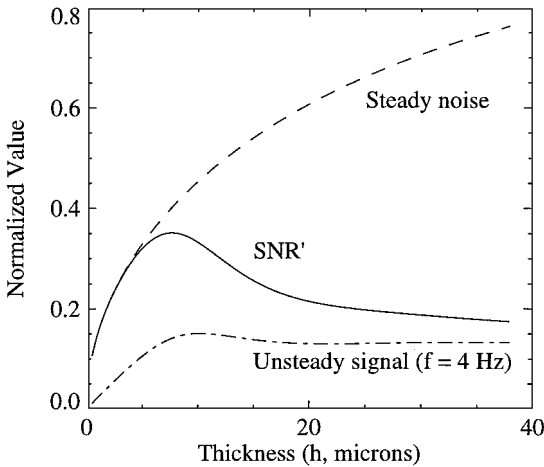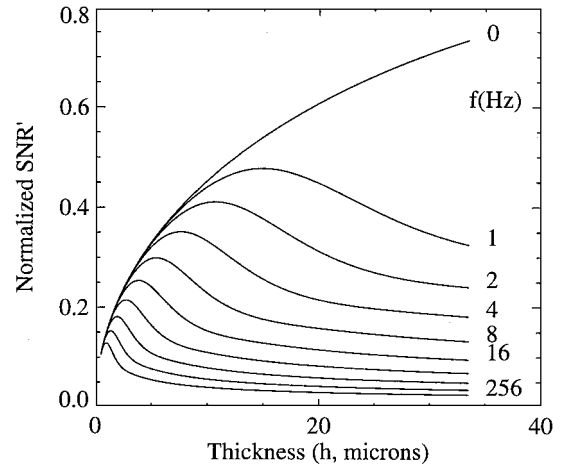

Fig. 6 Normalized SNR' vs thickness: $\delta/h = 0.01\,\mu\mathrm{m}^{-1}$ and $D = 10^3\ \mu\mathrm{m}^2/\mathrm{s}$.
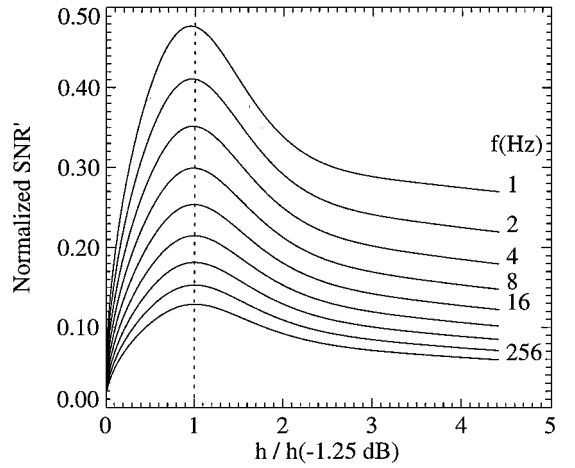
Fig. 7 Normalized SNR' vs thickness ratio: $\delta/h = 0.01\ \mu\mathrm{m}^{-1}$ and $D = 10^3\ \mu\mathrm{m}^2/\mathrm{s}$.

near 1 indicates a balance between the unsteady frequency and the time response of the paint at the optimum thickness. Although it is not shown here, the optimum thickness ratio is also largely independent of unit optical density and diffusivity over a wide range of these values typical of PSPs. Thus, the optimum paint thickness is that thickness that results in a 1.25-dB attenuation of the unsteady signal. This thickness is shown in Fig. 8 as a function of frequency with unit optical density (Fig. 8a) and diffusivity (Fig. 8b) as parameters. For frequencies greater than about 10 Hz, the thickness for 1.25-dB attenuation is largely independent of unit optical density (Fig. 8a). In contrast, the effect of diffusivity is large over the entire range shown (Fig. 8b). This shows that the way to increase the optimum thickness for a particular application is to increase the diffusivity of the binder, a result that is consistent with intuition.

Unsteady Response: Nonlinear Solutions

All of the results to this point have been computed using a small perturbation approximation [Eq. (5)] that linearizes the problem. More general, nonlinear solutions for instantaneous pressures measured by PSP, $p_{\mathrm{psp}}(t)$, were computed by numerically integrating Eq. (4). Solutions were computed as functions of time over a full cycle of the applied pressure, and harmonic analyses were applied to the corresponding PSP pressure time histories.

In the general case, $p_{\mathrm{psp}}(t)$ is not a simple sinusoid, describable by only an amplitude ratio and phase shift, but includes higher-order harmonics. For example, Fig. 9 shows $[p_{\mathrm{psp}}(t) - \bar{p}]/\bar{p}$ as a function of phase angle for a severely nonlinear case $p'/\bar{p} = 0.9$, $h/h_{-1.25\mathrm{dB}} = 4.4$, and $f = 256$ Hz. The dash–dot line is the applied pressure, which varies between ± 0.9 of the average pressure,

Fig. 5 Normalized SNR' shown for $f = 4$ Hz: $\delta/h = 0.01\ \mu\mathrm{m}^{-1}$ and $D = 10^3\ \mu\mathrm{m}^2/\mathrm{s}$.
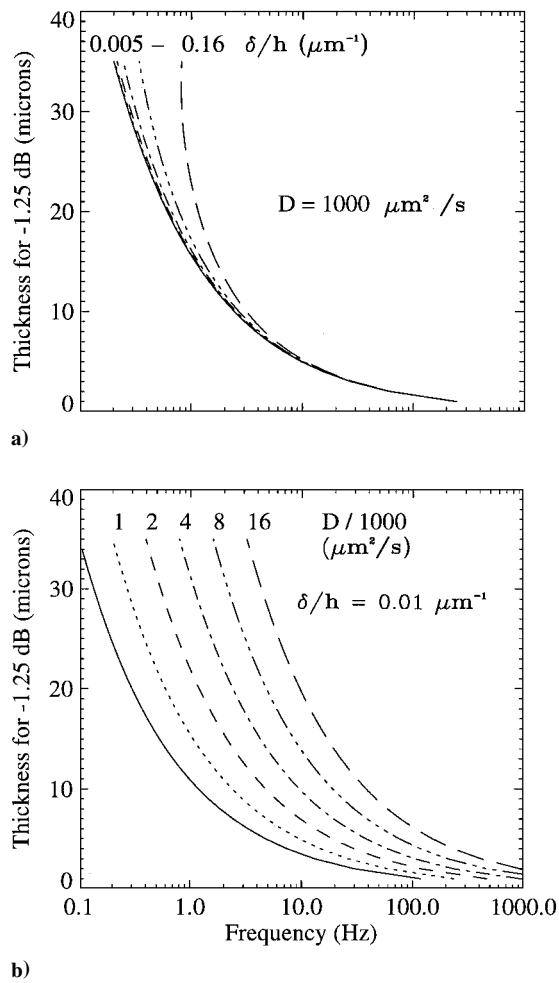
Fig. 8   Optimum thickness vs frequency for various a) unit optical densities and b) diffusivities.
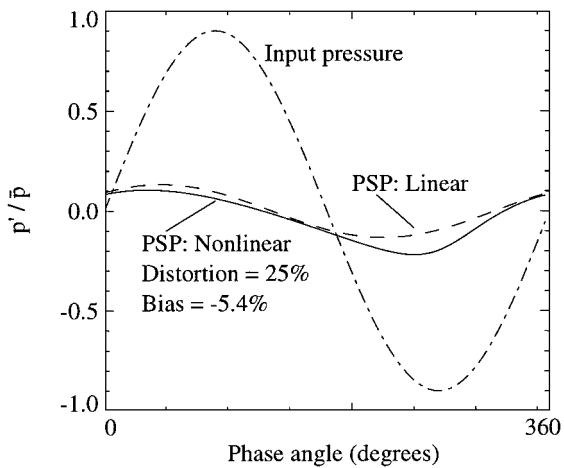

Fig. 9   Comparison of linear and nonlinear PSP solutions: thickness ratio = 4.4, $f = 256$ Hz, $\delta/h = 0.01$ μm^{-1}, and $D = 10^3$ μm^2/s.

a very large-amplitude unsteadiness, indeed. The dashed line is the small-perturbation solution; it is a sine wave of greatly diminished amplitude (given by the amplitude ratio r) and shifted phase. The mean value of the linearized solution is zero and, thus, equals the true mean unsteady pressure. The solid line is the nonlinear solution, which differs significantly from the small-perturbation solution in two ways. First, the signal is not a simple sine wave but includes higher-order harmonics and, thus, is distorted. This distortion makes it difficult to distinguish between higher-order harmonics that may exist in the applied unsteady pressure and harmonics introduced
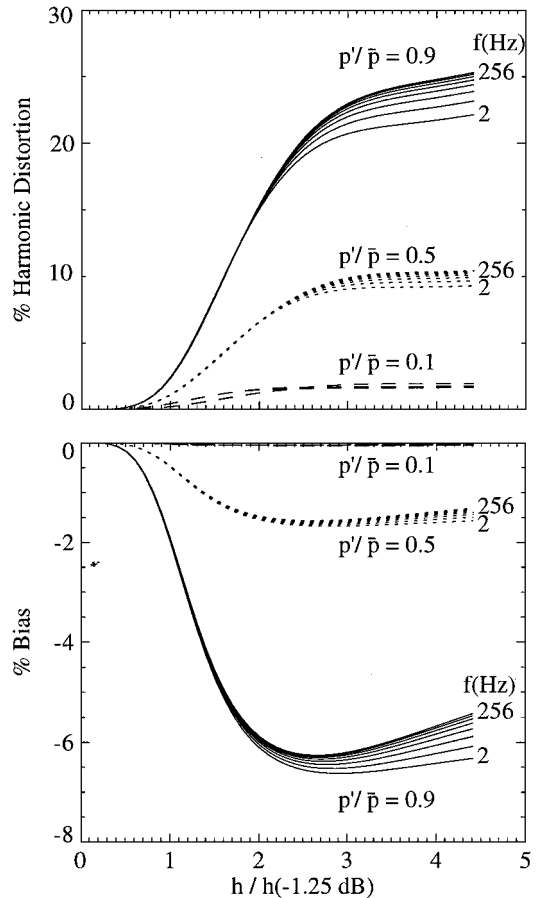

Fig. 10   With $\delta/h = 0.01$ μm^{-1}, $D = 10^3$ μm^2/s, $a = 0.2$, and $b = 0.8$: a) distortion and b) bias vs thickness ratio.

by the paint. Second, the nonlinear solution lies below the small-perturbation solution at all phase angles. Thus, the mean of instantaneous pressure measurements is negative and offset from the true mean.

The nonlinear effects can be quantified as percent harmonic distortion[16] (the square root of the sum of the squares of the amplitudes of the higher-order harmonics divided by the amplitude of the fundamental) and percent bias (the amplitude of the invariant first term of the Fourier series representation divided by the mean pressure). In the case shown in Fig. 9, distortion and bias are 25 and -5.4%, respectively. Although this is an extreme case, there are important applications where unsteady pressures can be very large compared to the mean. For example, the pressure near the leading edge of a helicopter rotor in forward flight undergoes very large changes during each cycle of rotation.

In Fig. 10, harmonic distortion and bias are shown as functions of paint thickness ratio with applied frequency and amplitude as parameters. For thickness ratios near 1.0, where paint thicknesses are near the optimum based on linear theory, distortion and bias are nearly independent of applied frequency and approximately scale with applied unsteady pressure amplitude $p'/\bar{p}$. Near the optimum, the distortion and bias (absolute value) will be less than about 3%. If the paint is significantly thicker than the optimum, however, very significant distortion and bias can occur. Note that the nonlinear example shown in Fig. 9 is the most severe case shown in Fig. 10 ($h/h_{-1.25\text{dB}} = 4.4$, $p'/\bar{p} = 0.9$, $f = 256$ Hz).

Discussion

Effect of a Base Coat

The preceding analysis does not account for the presence of a base coat, which is almost always applied between the pressure paint (referred to as the active layer in the subsequent discussion) and the model. The base coat would likely affect the response of the active layer in at least three ways. First, base coats are reflective and, thus, increase the optical thickness and signal brightness.

Second, the base coat may be permeable to oxygen, which alters the pneumatic thickness of the combined base coat and active layer. And third, luminophore may migrate into the base coat, where the oxygen diffusivity would, in general, differ from that in the active layer.[17]

Base coats may be described as black (nonreflecting at all wavelengths), red (reflecting only luminescence), blue (reflecting only excitation illumination), or white (reflecting both luminescence and excitation illumination). Consider, first, base coats that are impermeable to oxygen. A red base coat reflects luminescence that was initially directed toward the substrate and, thus, increases the steady signal intensity by a factor of up to 2.0 compared to a nonreflective (black) substrate [Eq. (1)]. A red base coat does not change either the frequency response or the optimum thickness of the active layer. It uniformly multiplies the emissive flux from all levels of the active layer and has the same effect as increasing the incident illumination.

A blue base coat increases signal intensity by reflecting unabsorbed excitation photons back through the active layer, resulting in additional absorption by and luminescence from the luminophore. This effect is zero for an optically thick active layer (because no incident photons penetrate to the base coat) and increases as the optical thickness of the layer decreases. The maximum signal amplification is 3.0 compared to a black basecoat[†] (perfectly reflective, Lambertian base coat, and infinitely thin active layer). The reflected excitation flux and, thus, the increased absorption and luminescence flux, are greatest at the bottom of the active layer and decrease exponentially with distance from the base coat. Therefore, although the signal is brighter than for a black base coat, the relative amplitude of the unsteady component is reduced. Additional analysis, which is beyond the scope of this paper, is required to establish how this affects SNR′ and optimum thickness.

A white base coat combines the properties of red and blue base coats. Thus, although the signal level can be significantly enhanced (by a factor of up to $2 \times 3 = 6$), the frequency response will decrease, especially for layers that are optically thin.

A base coat (black, red, blue, or white) that is permeable to oxygen increases the effective pneumatic thickness of the active layer and, thus, decreases its frequency response. This is always detrimental to SNR′ if the luminophore is confined to the active layer[†]; however, the effect on optimum thickness is not obvious. If the luminophore has migrated into the base coat, the base coat may either increase or decrease SNR′ depending on the thicknesses of the layers. This can be illustrated by the special case where the concentration of luminophore in the base coat and the diffusivity and solubility of the base coat to oxygen are the same as for the active layer, that is, the base coat is merely an extension of the active layer. In this case, if the combined thickness of the active layer plus base coat is more nearly equal to the optimum thickness, SNR′ will be greater than for the active layer alone.

Other Considerations

The size of the luminophore imposes a lower limit on the thickness of a paint layer. Paint layers that are thinner than this limit will be rough and could alter the development of boundary layers. This limit is particularly severe for binary pressure paints that include a reference, pressure-insensitive luminophore. To date, the best reference luminophores have been crystals rather than molecules with diameters up to 4–5 μm. To avoid this limit, researchers in Russia have developed a fast-responding pressure paint in which the reference luminophore is applied as a separate layer below the pressure-sensitive layer.[18] Because both layers are excited by light of the same frequency, the optical density of the pressure-sensitive layer must be low enough to allow adequate excitation of the reference sublayer. This effectively limits the concentration of luminophore in the pressure-sensitive layer.

Application of the results of this paper requires that a uniform coat of paint of known thickness is applied to the model. This is not easy to do with a spray gun, which is how most pressure paints are applied. Finally, the present analysis assumes that the concentration of luminophore is uniform through the depth of a paint layer. This is a good assumption for luminophores suspended in a binder. However, if the luminophore is adsorbed directly to the surface of the model, a technique that has recently been demonstrated to allow high-frequency response,[19] the assumption will generally not be valid.

Conclusions

There is a tradeoff between steady SNR ratio and unsteady frequency response as the thickness of a layer of PSP varies. An optimum thickness has been identified where the ratio of the unsteady signal amplitude to the steady noise is a maximum. This maximum occurs at the thickness corresponding to a 1.25-dB attenuation of the unsteady signal amplitude and decreases as the unsteady frequency increases. The result is valid over a wide range of unit optical densities and diffusivities characteristic of pressure paints. If the unsteady pressure is not small compared to the mean pressure, and if the paint is significantly thicker than optimum, then the unsteady PSP signal may be distorted by higher-order harmonics and offset from the true mean unsteady pressure.

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